Higher-order adaptivity techniques for modeling fast transients on high-voltage lines

P. Karban, V. Kotlan, I. Dolezel, *Members, IEEE*

University of West Bohemia, Faculty of Electrical Engineering, Univerzitní 26, 306 14 Pilsen, Czech Republic,
{karban, vkotlan, idolezel}@kte.zcu.cz

Adaptive techniques for modeling fast transients on high-voltage lines are proposed and evaluated. These techniques seem to be a powerful tool for their higher-order finite element analysis, bringing considerable savings both in memory and time of computation. A typical example is presented showing the advantage of this approach.

***Index Terms*—Finite element analysis, adaptive techniques, FEM, FDM, FDTD**

I. Introduction

NUMERICAL MODELING of fast transient phenomena (mostly of atmospheric or switching origin) on high-voltage lines is still a challenge. Although it is usually formulated as a 2D task (one spatial coordinate and time), its solution may take a long time and its convergence is often rather poor. Presently, perhaps the most popular for its solution is the finite-difference time domain method (FDTD) [1], [2].

II. Adaptive techniques

The paper presents an alternative approach based on a fully adaptive higher-order finite element method (*hp*-FEM). The line can generally be ended by any RLC load.

First, an adaptive method was written in time for a fixed spatial mesh, that is based on using both explicit and implicit Runge-Kutta methods. The fundamental drawback of these methods is the necessity of a great amount of the time steps, which strongly depends on the spatial mesh. This problem is also well-known from the finite-difference approach; a finer mesh does not generally imply a better solution (using very fine meshes may lead to a very poor convergence or oscillations). In the case of *hp*-FEM, this effect is even amplified by the orders of the corresponding polynomials.

As for the p and h variants, difficulties still occur in connection with the explicit time methods because of their poor convergence. This may be caused, however, by general properties of explicit methods for coarser meshes, where they lose their stability.

III. Numerical solution

The problem is solved using own application Agros [3], [4] based on a fully adaptive higher-order finite element method in cooperation with the library deal.II [5], [6]. The adaptive solution is carried out by the Runge-Kutta methods, for which the equations are transformed to the form

$$\frac{\partial f\left(x,t\right)}{\partial t} = q\left(t, f\left(x,t\right)\right) \, . \tag{1}$$

After successive multiplying this equation by the test functions, integrating over the domain Ω and several more operation we get the spatially discretized (1) in the form

$$\boldsymbol{M}\frac{\mathrm{d}\,\boldsymbol{U}}{\mathrm{d}\,t} = \boldsymbol{q}\left(t,\boldsymbol{U}\right) \, , \tag{2}$$

where the function $\boldsymbol{q}\left(t,\boldsymbol{U}\right)$ is the discretized form of the function $q\left(t, f\left(x,t\right)\right)$ and $\boldsymbol{M}$ is the mass matrix.

The algorithm also includes the transfer of solution in particular time steps to new integration points. It also allows setting the number of steps after which the mesh is either refined or made coarser. This also positively contributes to the velocity of computations.

IV. Illustrative example

The following example illustrates the methodology on a one-phase overhead $22\,\mathrm{kV}$ line of length $1000\,\mathrm{m}$. The line is loaded by a surge voltage wave of the standard shape (8/20) (see Fig. 1) that is expressed by the formula

$$u(t) = U_{\max}\,\exp^{-63300t}\sin(109000t)\, . \tag{3}$$

The line is ended by the Neumann condition.

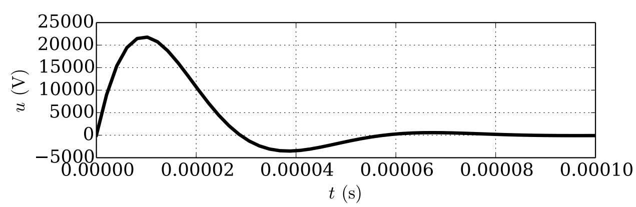

Fig. 1. Standard voltage wave for switching procceses

The solution is carried out using the singly-diagonally-implicit Runge-Kutta (SDIRK) method. The equations for voltage u and current i are given in the form

$$\frac{\partial i}{\partial x} = -C'\frac{\partial u}{\partial t} - G'u\, , \tag{4}$$

$$\frac{\partial u}{\partial x} = -L'\frac{\partial i}{\partial t} - R'i\, , \tag{5}$$

where $R' = 56.8 \times 10^{-6}\,\Omega \cdot \mathrm{m}^{-1}$, $L' = 1.6 \times 10^{-6}\,\mathrm{H} \cdot \mathrm{m}^{-1}$, $C' = 6.94 \times 10^{-9}\,\mathrm{F} \cdot \mathrm{m}^{-1}$, and $G' = 0$ are the resistance, inductance, capacitance and conductance per unit length, respectively. The conductance was not considered in the model.

V. Results

At the beginning of computations, a relatively fine mesh is generated, but its successive coarsening quickly leads to the optimal number of the degrees of freedom (DOFs). The preset values follow: the number of time steps $n = 200$ and the final time $t_{\mathrm{stop}} = 10^{-4}$ s.

A. *h-adaptivity*

First, the task was solved using the *h*-adaptivity. The elements were of the second order ($p = 2$) and the value of the initial global refinement was 10 (this value denotes the level of initial uniform refinement of the mesh). Figure 2 shows the distribution of voltage along the line at time $t = 4 \times 10^{-5}$ s, the number of DOFs being 222.

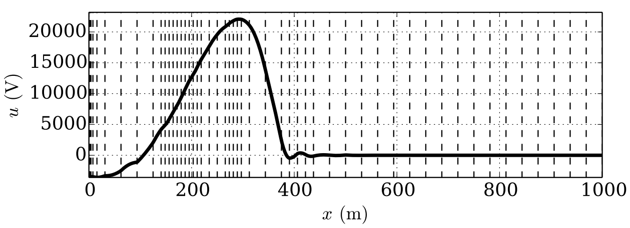

Fig. 2. Distribution of voltage along the line at time $t = 4 \times 10^{-5}$ s (*h*-adaptivity, 222 DOFs)

B. *p-adaptivity*

Next, the same task was solved using the *p*-adaptivity. At the beginning, the elements were of the second order and the value of the initial global refinement was 7 (here the level of fineness is lower because the *p*-adaptivity modifies the mesh substantially faster). Figure 3 shows an analogous distribution of voltage along the line at time $t = 6 \times 10^{-5}$ s, the number of DOFs being 230.

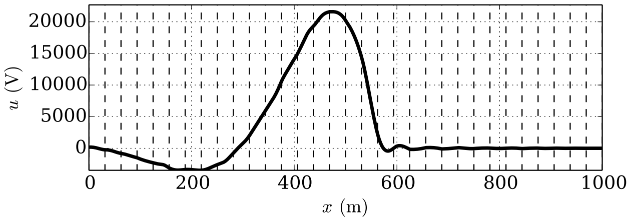

Fig. 3. Distribution of voltage along the line at time $t = 6 \times 10^{-5}$ s (*p*-adaptivity, 230 DOFs)

For this kind of adaptivity, Fig. 4 shows the distribution of the orders of polynomials for the same time $t = 6 \times 10^{-5}$ s.

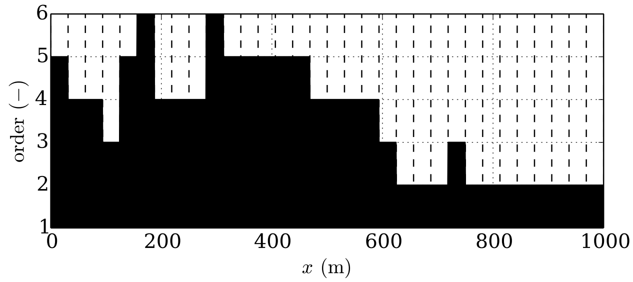

Fig. 4. Distribution of the orders of polynomials in the elements along the line at time $t = 6 \times 10^{-5}$ s (*p*-adaptivity, 230 DOFs)

C. *Comparison*

Figure 5 shows the time evolution of DOFs for both variants of adaptivity. Obviously, the *p*-adaptivity is much favorable, the corresponding values at the beginning of the process are lower almost by an order.

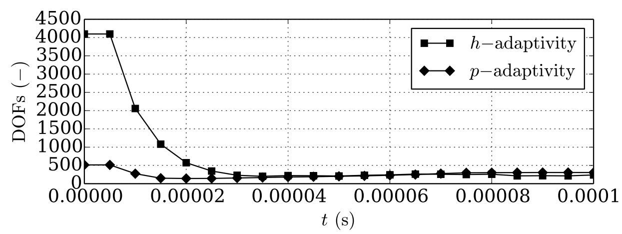

Fig. 5. Time evolution of the number of DOFs

VI. Conclusion

The full paper will contain a comparison of parameters of computation for the *hp*-FEM and Backward Differentiation Formula-based method, both including the adaptive techniques. The comparison will be performed for more complicated examples including three-phase models and non-symmetries.

Acknowledgement

This work was supported by the European Regional Development Fund and Ministry of Education, Youth and Sports of the Czech Republic (project No. CZ.1.05/2.1.00/03.0094: Regional Innovation Centre for Electrical Engineering - RICE) and Grant project GACR P102/11/0498.

References

[1] A. Taflove, “Advances in FTDT Computational Electrodynamics: Photonics and Nanotechnology,” Artech House, Boston, 2013.
[2] Z. Benešová, V. Kotlan, “Propagation of Surge Waves Induced by Lightning Stroke on Interconnected Transmission Lines,” in *Acta Technica,* Volume 51, Issue 4, pp. 301–316.
[3] P. Karban, F. Mach, P. Kus, D. Panek, I. Dolezel, “Numerical Solution of Coupled Problems Using Code Agros2D,” in *Computing,* Volume 95, Issue 1 Supplement, 2013, pp. 381–408.
[4] P. Karban, F. Mach, I. Dolezel, “Advanced Adaptive Algorithms in 2D Finite Element Method of Higher Order of Accuracy,” in *COMPEL,* Volume 32, Issue 3, 2013, pp. 834–849.
[5] W. Bangerth, T. Heister, L. Heltai, G. Kanschat, M. Kronbichlerand, M. Maier, B. Turcksin, T. D. Young, “The deal.II Library, Version 8.1,” *arXiv preprint, http://arxiv.org/abs/1312.2266v4*, 2013.
[6] W. Bangerth, R. Hartmann, G. Kanschat, “deal.II – a General Purpose Object Oriented Finite Element Library,” in *ACM Trans. Math. Softw.,* Volume 33, Issue 33, 2007, pp. 24/1–24/27.